

**Final Exam MTH 211 Fall 2010**

Ayman Badawi

**QUESTION 1. (Each is 4 points, Total = 64)**

- (i) The measurement of each interior angle of a regular 10-gon is  
a) 36 (b) 144 (c) 100 (d) 108
- (ii) The measurement of each center angle of a regular 15-gon is  
a) 156 (b) 12 (c) 24 (d) 225
- (iii) One of the following is constructible by unmarked ruler and a compass:  
a) regular 21-gon (b) regular 22-gon (c) regular 34-gon (d) regular 50-gon
- (iv) Given  $C$  is a circle centered at  $O$  and with radius 6 cm. Let  $A$  be a point such that  $|OA| = 3$ . Let  $Inv(A)$  be the inversion of  $A$  with respect to  $C$ . Then  $|OInv(A)| =$   
a) 2 (b) 12 (c) 9 (d) 4.5
- (v) If a regular  $n$ -gon is constructible, then the angle  $(180/n)$  is constructible.  
a) True (b) False
- (vi) If an angle  $\alpha$  is constructible, then the angle  $\alpha/16$  is constructible.  
a) True (b) False
- (vii) Let  $C$  be a circle centered at  $O$  and with radius 3. Given  $A$  is a point such that  $|OA| = 1$  and  $D$  is a circle orthogonal to  $C$  and passing through  $A$ . Then one of the following values is a possibility for the radius of  $D$ :  
a) 3 (b) 5 (c) 3.5 (d) 2
- (viii) Let  $H$  be the horizon circle (the model for non-Euclidean) with radius 4 and centered at  $O$ . Let  $A$  be a point in  $H$  such that  $|OA| = 3$ . Then the non-Euclidean distance between  $O$  and  $A$  is :  
a)  $\ln(3)$  (b)  $\ln(7)$  (c)  $\ln(9) = 2\ln(3)$  (d)  $\ln(4)$
- (ix) In non-Euclidean (hyperbolic) geometry, if  $a, b$  are two points, then  
a) There are infinitely many lines pass through  $a$  and  $b$  (b) There is exactly one circle passes through  $a$  and  $b$   
c) There is exactly one line passes through  $a$  but not through  $b$  (d) There is exactly one line passes through  $a$  and  $b$ .
- (x) In non-Euclidean Geometry, the sum of all interior angles of a regular 4-gon is  
a) 180 (b) less than or equal to 180 (c) 360 (d) less than 360
- (xi) One of the below is a possibility for the inversion of the arc  $ab$  with respect to the circle  $C$  (the arc  $ab$  is a part of a circle not passing through the center of  $C$ )

(xii) One of the below is a possibility for the inversion of the arc  $ab$  with respect to the circle  $C$  (the arc  $ab$  is a part of a circle passing through the center of  $C$ )

(xiii) Let  $C$  be a circle with radius 4 and centered at  $O$ . Let  $Q$  be a point on  $C$ . Draw a circle call it  $D$  centered at  $Q$  with radius 4 again (note that  $D$  passes through  $O$ ). The two circles intersect in two points, say  $A$  and  $B$ . Now choose a point say  $Z$  on  $D$  such that the line segment  $OZ$  is a diameter of  $D$ . Now the line segment  $AB$  intersects the diameter  $OZ$  in a point say  $M$  (note that  $AB$  is perpendicular to  $OZ$ ). The inversion of  $M$  with respect to the circle  $C$  is

a) the point  $Z$    b) a point outside the circle  $D$    c) a point outside  $C$  but inside  $D$  and not on  $D$    d) is the mid point of the line segment  $QZ$ .

(xiv) In the previous question, the length of  $AZ$  is

a) 4   b)  $4\sqrt{3}$    c) 6   d)  $2\sqrt{3}$

(xv) The length of  $AQ$  in question XIII is

a) 2   b)  $\sqrt{2}$    c)  $2\sqrt{3}$    d)  $4\sqrt{3}$

(xvi) Let  $K$  be the mid-point of the line segment  $OM$  as in question XIII. The inversion of  $K$  with respect to  $C$  is

a) a point inside  $D$  but outside  $C$    b) the mid-point meter  $OZ$    c) the mid-point of  $QZ$    d) a point outside  $D$  but on the line extension of  $OZ$

**QUESTION 2. (12 points)** Let  $H$  be a horizon circle (a model for non-Euclidean geometry) centered at  $O$  and with radius 4. Construct a non-Euclidean triangle inside  $H$  call it  $OAB$  such that  $|OA| = |OB| = 2$  and  $OA$  is perpendicular to  $OB$ . (Note that  $|OA|$  indicates the Euclidean distance between  $A$  and  $O$ ). **OUTLINE THE STEPS BY STATING THE CRUCIAL STEPS IN THE CONSTRUCTION.**

Use a marked ruler in order to find the NON-EUCLIDEAN DISTANCE between  $A$  and  $B$  (You may measure to the nearest decimal)

**QUESTION 3. (12 points)** Draw a horizontal line and call it  $L_1$ , draw another line and call it  $L_2$  such that  $L_2$  intersects  $L_1$  at an angle 90 degrees. Let  $C$  be a point that does not lie on either  $L_1$  nor  $L_2$ . Find two points say  $a$  on  $L_2$  and  $b$  on  $L_1$  such that  $C$  lies on the line segment  $ab$  and  $|ac| = 1.5 |cb|$ . OUTLINE THE STEPS BY STATING THE CRUCIAL STEPS IN THE CONSTRUCTION.

**QUESTION 4. (12 points)** Let  $ab$  be a diameter of a semicircle. Find two points say  $D, F$  lying on the arc of the semicircle and two points say  $X, Y$  lying on the diameter  $ab$  such that  $DFXY$  is a rectangle with  $FX$  as the length,  $XY$  as the width, and  $|FX| = 2|XY|$ . **OUTLINE THE STEPS BY STATING THE CRUCIAL STEPS IN THE CONSTRUCTION.**

#### Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.

E-mail: [abadawi@aus.edu](mailto:abadawi@aus.edu), [www.ayman-badawi.com](http://www.ayman-badawi.com)